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OPTIMAL CONTROL MODEL FOR THE TRANSMISSION OF DENQUE FEVER

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ABSTRACT
In this work an SEIR model that described the dynamics of transmission of denque fever were proposed and analyzed. The standard method is used to analyze the behaviors of the proposed model. The results shown that there were two equilibrium points; disease free and endemic equilibrium point. The qualitative results are depended on a basic reproductive number . We obtained the basic reproductive number by using the next generation method and finding the spectral radius. Routh-Hurwitz criteria is used for determining the stabilities of the model. If , then the disease free equilibrium point is local asymptotically stable: that is the disease will died out, but if , then the endemic equilibrium is local asymptotically stable. After that the SEIR model is modified from the first model by adding optimal control functions that includes two time – dependent control functions with one minimizing the contract between the susceptible human and the infected vector and the other, minimizing the population of the infected human. The result from the numerical solutions of the models are shown and compared for supporting the analytic results.

KEYWORD: Denque ferver , stability, equilibrium point, basic reproductive number, optimal control model..

INTRODUCTION
Dengue is a mosquito-borne infection found in tropical and sub-tropical regions around the world. In recent years, transmission has increased predominantly in urban and semi-urban areas and has become a major international public health concern.Severe dengue (also known as Dengue Haemorrhagic Fever) was first recognized in the 1950s during dengue epidemics in the Philippines and Thailand. Today, severe dengue affects most Asian and Latin American countries and has become a leading cause of hospitalization and death among children in these regions (WHO) [1]. estimated about 50 to 100 million cases reported. Around 500,000 people are estimated to be infected by hemorrhagic dengue fever each year. Four serotypes of the dengue virus exist and are called DEN1, DEN2, DEN3 and DEN4. Infection by one type of the virus confirms permanent immunity to further infections by the infecting strain and temporary immunity to the others. The disease is usually found in tropical region of the world. This disease can be transmitted to human by biting of infected *Aedes Aegypti* mosquitoes [1]. This mosquito breeds in artificial water containers such as discarded tires, cans, barrels and flower vases, all of which are usually found in the domestic environment. Dengue fever (DF) is characterized by the rapid development of the illness that may last from five to seven days with headache, joint and muscle pain and a rash [2]. The symptom of dengue patients may occur from four to twelve days after exposure to an infected mosquito. Current data suggests that the immune Mathematical models have become an important tool for understanding the spread and control of disease. Because of this disease is caused by virus, therefore no drug can cure this disease specificially. This paper is organized as follows. In section 2, we present an SEIR model for denque fever in Andaman area The standard method is used to analyze the behaviors of the proposed model. The analysis of optimization problem is presented in section 3. In section4, we give a numerical appropriate method and the simulation corresponding results. Finally, the conclusions are summarized in section 5.

MATHEMATICAL MODEL
In this paper, we study the transmission of denque fever through mathematical modeling. By using the standard method to analyze the behaviours of the proposed model which was adopted [2]. The population consist of two groups : human population N_h and population vector N_υ . Human population divided into four disease-state compartments: susceptible individuals $\left(\overline{S}\right)$, people who can catch the disease; exposed individuals $\left(\overline{E}\right)$, people

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whose body is a host for the infectious agent but are not yet able to transmit the disease; infectious (infective) individuals $\left(\bar{I}\right)$, people who have the disease and can transmit the disease; recovered individuals $\left(\bar{R}\right)$, people who have recovered from the disease. Population vector or mosquitoes N_υ are divided into three groups of mosquitoes: the susceptible mosquitoes population $\overline{S_\upsilon}$, the exposed mosquitoes population $\overline{E_\upsilon}$ and the infected mosquitoes population $\overline{I_\upsilon}$. In this study, we assumed that there are numbers of people in the populations that have already infected by the virus while others have not. It is also assumed that the transmission of the virus continues in the population but number of mosquitoes as the vector is constant. People and mosquitoes are categorized in one group at a time. Then we obtained the transmission model as shown by a system of ordinary differential equations as follows.

Human Population;

$$\frac{d\bar{S}}{dt} = \mu_h N_h - \left(\frac{\beta_h c \bar{I}_\upsilon}{N_h} + \rho + \mu_h\right)\bar{S} \tag{1}$$

$$\frac{d\bar{E}}{dt} = \left(\frac{\beta_h c \bar{I}_\upsilon}{N_h} + \rho\right)\bar{S} - (\mu_h + \varphi_h)\bar{E} \tag{2}$$

$$\frac{d\bar{I}}{dt} = \varphi_h \bar{E} - (\mu_h + \gamma_h + \alpha_h)\bar{I} \tag{3}$$

$$\frac{d\bar{R}}{dt} = \gamma_h \bar{I}_h - \mu_h \bar{R}_h \tag{4}$$

Where; $\bar{S} + \bar{E} + \bar{I} + \bar{R} = N_h$ (5)

Vector Population;

$$\frac{d\bar{S}_\upsilon}{dt} = B - \left(\frac{\beta_\upsilon c I_h}{N_h} + \mu_\upsilon\right)\bar{S}_\upsilon \tag{6}$$

$$\frac{d\bar{E}_\upsilon}{dt} = \left(\frac{\beta_\upsilon c I_h}{N_h}\right)\bar{S}_\upsilon - (\mu_\upsilon + \delta_\upsilon)\bar{E}_\upsilon \tag{7}$$

$$\frac{d\bar{I}_\upsilon}{dt} = \delta_\upsilon \bar{E}_\upsilon - \mu_\upsilon \bar{I}_\upsilon \tag{8}$$

Where; $\overline{S_\upsilon} + \overline{E_\upsilon} + \overline{I_\upsilon} = N_\upsilon = \frac{A}{\mu_\upsilon}$ (9)

Where;

$\bar{S}(t)$ is the susceptible human population at time t
$\bar{E}(t)$ is the exposed human population at time t
$\bar{I}(t)$ is the infected human population at time t
$\bar{R}(t)$ is the recovered human population at time t
N_h is the total number of human population ,
$\bar{S}_\upsilon(t)$ is the susceptible mosquitoes population at time t
$\bar{E}_\upsilon(t)$ is the exposed mosquitoes population at time t
$\bar{I}_\upsilon(t)$ is the infected mosquitoes population at time t
N_υ is the total number of mosquitoes population ,
c is average bite of mosquitoes that potentially infected
B is the constant recruitment rate of the mosquito,
μ_h is the death rate of human population ,
μ_υ is the death rate of mosquitoes population ,

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α_h is the death rate caused by denque itself

γ_h is the duration of infection in the body

δ_υ is the proportional rates of mosquitoes exposed to the virus infection

φ_h is the proportional rates for people exposed to the denque virus infection

$\frac{\beta_\upsilon c I_h}{N_h}$ is the rate of mosquitoes infected with denque virus

$\beta_\upsilon c = \gamma_\upsilon$ is the sufficient rate of correlation from vector to human

$\frac{\beta_\upsilon c I_\upsilon}{N_h}$ is the infection rate to individuals who have the potential to be infected

$\beta_h c$ is the sufficient rate of correlation from human to the vector

ρ is the percentage of infected mosquitoes

From equations (1) – (9) ,the model can be simplified as follows

$$\frac{d\bar{S}}{dt} = \mu_h N_h - (\frac{\beta_h c \bar{I}_\upsilon}{N_h} + \rho + \mu_h)\bar{S} \qquad (10)$$

$$\frac{d\bar{E}}{dt} = (\frac{\beta_h c \bar{I}_\upsilon}{N_h} + \rho)\bar{S} - (\mu_h + \varphi_h)\bar{E} \qquad (11)$$

$$\frac{d\bar{I}}{dt} = \varphi_h \bar{E} - (\mu_h + \gamma_h + \alpha_h)\bar{I} \qquad (12)$$

$$\frac{d\bar{E}_\upsilon}{dt} = (\frac{\beta_\upsilon c \bar{I}}{N_h})\bar{S}_\upsilon - (\mu_\upsilon + \delta_\upsilon)\bar{E}_\upsilon \qquad (13)$$

$$\frac{d\bar{I}_\upsilon}{dt} = \delta_\upsilon \bar{E}_\upsilon - \mu_\upsilon \bar{I}_\upsilon \qquad (14)$$

We normalize equations (10)-(14) by letting $S = \frac{\bar{S}}{N_h}$, $E = \frac{\bar{E}}{N_h}, I = \frac{\bar{I}}{N_h}, E_\upsilon = \frac{\bar{E_\upsilon}}{N_\upsilon} and I_\upsilon = \frac{\bar{I_\upsilon}}{N_\upsilon}$ and ,then we get;

The system of equations (15):

$$\begin{aligned}
\frac{dS}{dt} &= \mu_h(1-S) - \rho S - \alpha_h S I_\upsilon \\
\frac{dE}{dt} &= (\alpha_h I_\upsilon + \rho)S - (\mu_h + \varphi_h)E \\
\frac{dI}{dt} &= \varphi_h E - (\mu_h + \gamma_h + \alpha_h)I \qquad (15) \\
\frac{dE_\upsilon}{dt} &= \gamma_\upsilon(1 - I_\upsilon - E_\upsilon)I - (\mu_\upsilon + \delta_\upsilon)E_\upsilon \\
\frac{dI_\upsilon}{dt} &= \delta_\upsilon E_\upsilon - \mu_\upsilon I_\upsilon
\end{aligned}$$

Where R(t) , S_υ can be obtained from equations $S(t)+E(t)+I(t)+R(t)=1$ and $S_\upsilon + E_\upsilon + I_\upsilon = 1$

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Basic properties of the model

The equilibrium points for $(S^*, E^*, I^*, E_\upsilon^*, I_\upsilon^*)$ are found by setting the right hand side of each equations (15) equal to zero. We obtained two equilibrium points as follows;

$$S = \frac{\mu_h}{(\mu_h + \rho - \alpha_h I_\upsilon)},$$

$$E = \frac{(\alpha_h I_\upsilon + \rho)\mu_h}{(\mu_h + \varphi_h)(\mu_h + \rho - \alpha_h I_\upsilon)},$$

$$I = \frac{(\mu_h \varphi_h)(\alpha_h I_\upsilon + \rho)}{(\mu_h + \gamma_h + \alpha_h)(\mu_h + \varphi_h)(\mu_h + \rho - \alpha_h I_\upsilon)},$$

$$E_\upsilon = \frac{\gamma_\upsilon(\mu_h \varphi_h)(\alpha_h I_\upsilon + \rho)}{\gamma_\upsilon(\mu_h \varphi_h)(\alpha_h I_\upsilon + \rho) + (\mu_\upsilon + \delta_\upsilon)(\gamma_\upsilon I + \mu_\upsilon + \delta_\upsilon)(\mu_h + \gamma_h + \alpha_h)(\mu_h + \varphi_h)(\mu_h + \rho - \alpha_h I_\upsilon)},$$

$$I_\upsilon = \frac{\delta_\upsilon \gamma_\upsilon(\mu_h \varphi_h)(\alpha_h I_\upsilon + \rho)}{\mu_\upsilon(\gamma_\upsilon(\mu_h \varphi_h)(\alpha_h I_\upsilon + \rho) + (\mu_\upsilon + \delta_\upsilon)(\gamma_\upsilon I + \mu_\upsilon + \delta_\upsilon)(\mu_h + \gamma_h + \alpha_h)(\mu_h + \varphi_h)(\mu_h + \rho - \alpha_h I_\upsilon))}$$

Disease Free Equilibrium Point (E_0) : In the absence of the disease in the community, there are $I = 0$ and $I_\upsilon = 0$, we obtained $E_0(S, E, I, E_\upsilon, I_\upsilon)$ where

$$S = \frac{\mu_h}{(\mu_h + \rho)}, E = \frac{\rho\mu_h}{(\mu_h + \varphi_h)(\mu_h + \rho)}, I = 0,$$

$$E_\upsilon = \frac{\gamma_\upsilon(\mu_h \varphi_h)\rho}{\gamma_\upsilon(\mu_h \varphi_h)\rho + (\mu_\upsilon + \delta_\upsilon)(\mu_\upsilon + \delta_\upsilon)(\mu_h + \gamma_h + \alpha_h)(\mu_h + \varphi_h)(\mu_h + \rho)}, I_\upsilon = 0$$

Endemic Equilibrium Point (E_1) **:** In case the disease is presented in the community, $I \neq 0 and I_\upsilon \neq 0$, we obtained, $E_1(S^*, E^*, I^*, E^*_\upsilon, I^*_\upsilon)$ where;

$$S^* = \frac{\mu_h}{(\mu_h + \rho - \alpha_h I^*_\upsilon)},$$

$$E^* = \frac{(\alpha_h I^*_\upsilon + \rho)\mu_h}{(\mu_h + \varphi_h)(\mu_h + \rho - \alpha_h I^*_\upsilon)},$$

$$I^* = \frac{(\mu_h \varphi_h)(\alpha_h I^*_\upsilon + \rho)}{(\mu_h + \gamma_h + \alpha_h)(\mu_h + \varphi_h)(\mu_h + \rho - \alpha_h I^*_\upsilon)},$$

$$E^*_\upsilon = \frac{\gamma_\upsilon(\mu_h \varphi_h)(\alpha_h I^*_\upsilon + \rho)}{\gamma_\upsilon(\mu_h \varphi_h)(\alpha_h I^*_\upsilon + \rho) + (\mu_\upsilon + \delta_\upsilon)(\gamma_\upsilon I^* + \mu_\upsilon + \delta_\upsilon)(\mu_h + \gamma_h + \alpha_h)(\mu_h + \varphi_h)(\mu_h + \rho - \alpha_h I^*_\upsilon)},$$

$$I_\upsilon = \frac{\delta_\upsilon \gamma_\upsilon(\mu_h \varphi_h)(\alpha_h I^*_\upsilon + \rho)}{\mu_\upsilon(\gamma_\upsilon(\mu_h \varphi_h)(\alpha_h I^*_\upsilon + \rho) + (\mu_\upsilon + \delta_\upsilon)(\gamma_\upsilon I^* + \mu_\upsilon + \delta_\upsilon)(\mu_h + \gamma_h + \alpha_h)(\mu_h + \varphi_h)(\mu_h + \rho - \alpha_h I^*_\upsilon))}$$

Basic Reproductive Number (R_0)

We obtained a basic reproductive number by using the next generation method (van den Driessche and Watmough, 2002)[9]. By rewriting the equations $(5)-(7)$ in matrix form ;

$$\frac{dX}{dt} = F(X) - V(X) \qquad (16)$$

Where $F(X)$ is the non-negative matrix of new infection terms and $V(X)$ is the non-singular matrix of remaining transfer terms.

And setting;

$$F = \left[\frac{\partial F_i(E_0)}{\partial X_i}\right] \text{ and } V = \left[\frac{\partial V_i(E_0)}{\partial X_i}\right] \qquad (17)$$

for all $i, j = 1,2,3,4,5$, be the Jacobean matrix of $F(X)$ and $V(X)$ at E_0 .The basic reproductive number (R_0) is the number of secondary case generate by a primary infectious case (Andeson and May, 1991; van den Driessche

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and Watmough, 2002) or basic reproductive number is a measure of the power of an infectious disease to spread in a susceptible population. It can be evaluated through the formula;

$$\rho\left(FV^{-1}\right). \qquad (18)$$

Where FV^{-1} is called the next generation matrix and $\rho\left(FV^{-1}\right)$ is the spectral radius (largest eigenvalues) of FV^{-1}. Then we get the reproduction number R_0 where ,

$$R_0 = \frac{(\beta_h c I_\upsilon / N_h) S_0}{\gamma_h} \qquad (19)$$

Finally, Routh-Hurwitz criteria is used for determining the stabilities of the model. . If $R_0 < 1$, then the disease free equilibrium point is local asymptotically stable: that is the disease will died out, but if $R_0 > 1$, then the endemic equilibrium is local asymptotically stable. Optimal control is the standard method for solving dynamic optimization problems ,when those problems are expressed in continuous time (Lenhart and workman,2006). In this paper ,we use this method as part of control measures for denque fever epidemics. Into the system of equations (15), we include two controls *a* and *b* that represent, respectively, the effort that reduces the contract between the infectious vector and the susceptible individuals and also to reduces the infectious human. The mathematical system with controls is given by the nonlinear differential equations subject to non-negative initial conditions as the following;

$$\begin{aligned}
\frac{dS}{dt} &= \mu_h(1-S) - \rho S - \alpha_h(1-a) S I_\upsilon \\
\frac{dE}{dt} &= (\alpha_h I_\upsilon)(1-a) S + \rho S - (\mu_h + \varphi_h) E \\
\frac{dI}{dt} &= \varphi_h E - (\mu_h + \gamma_h + \alpha_h) b I \\
\frac{dE_\upsilon}{dt} &= \gamma_\upsilon (1 - I_\upsilon - E_\upsilon) I b - (\mu_\upsilon + \delta_\upsilon) E_\upsilon \\
\frac{dI_\upsilon}{dt} &= \delta_\upsilon E_\upsilon - \mu_\upsilon I_\upsilon
\end{aligned} \qquad (20)$$

With value $\alpha_h = \dfrac{\beta_h c B}{N_h \mu_\upsilon}$,

$S(0) \geq 0, E(0) \geq), I(0) \geq 0, E_\upsilon(0) \geq 0$ and $I_\upsilon(0) \geq 0$

OPTIMAL CONTROL FOR THE DYNAMICS OF DENQUE FEVER MODEL

In this section we use the optimal control theory to analyze the behavior of the system of equations (20) . The objective one is to minimize the susceptible human and the infected vector and the other is to minimize the population of the infected human. Mathematically, for a fixed terminal time t_f , the problem is to minimize the objective functional**;**

$$J(a,b) = \int_0^{t_f} \left[E(t) + I(t) + \frac{B_1}{2} a^2(t) + \frac{B_2}{2} b^2(t) \right] dt \qquad (21)$$

The parameter $B_1 \geq 0$ and $B_2 \geq 0$ denote weights that balance the size of the terms for a fixed terminal time t_f . Hence we are interested in finding an optimal control pair a^* and b^* ,such that:

$$J\left(a^*, b^*\right) = \min\left\{ J(a,b) : (a,b) \varepsilon U \right\} \qquad (22)$$

Where, $U = \left\{ (a,b) : 0 \leq a, b \leq 1, t \in \left[0, t_f\right], \text{u } a \text{and } b \text{ are } \textit{Lebesgue measurable} \right\}$

Next, applying the Pontryagin's Maximum Principle (Kirschner et al.,1997),we derive necessary conditions for our optimal control and corresponding state variables, including the two control functions. Therefore we have five

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corresponding adjoint variables where λ_1 corresponds to S , λ_2 corresponds to E , λ_3 corresponds to I , λ_4 corresponds to E_υ and λ_5 corresponds to I_υ.

The Hamiltonian adjoint equations

The Hamiltonian equation is formed by allowing each of the adjoint variables to correspond to each of the state variables accordingly and combining the result with the objective functional as below:

$$H = E(t) + I(t) + \frac{B_1}{2}a^2(t) + \frac{B_2}{2}b^2(t) + \sum_{i=1}^{5}\lambda_i f_i \qquad (23)$$

Where f_i is the right hand side of the differential equation of the i^{th} state variables.

The adjoint equations are formed by taking the derivative of the Hamiltonian with respect to each of the state variables as follow;

By applying the Pontryagin's maximum principle [9] and the existence result of optimal control from [10], we obtain the following theorem:

Theorem 1

There exists an optimal control $(a^*, b^*) \in U$,and corresponding solution $S^*, E^*, I^*, E^*_\upsilon and I^*_\upsilon$ that minimize $J(a,b)$ over U .And there exists adjoint functions $\lambda_1, \lambda_2, \lambda_3, \lambda_4 and \lambda_5$ verifying;

$$\lambda_1' = -\frac{\partial H}{\partial S} = \lambda_1(\mu_h + \rho + \alpha_h(1-a(t))I_\upsilon) - \lambda_2((\alpha_h I_\upsilon)(1-a(t)) + \rho) \ ,$$

$$\lambda_2' = -\frac{\partial H}{\partial E} = \lambda_2(\mu_h + \varphi_h) - \lambda_3\varphi_h - 1 \ ,$$

$$\lambda_3' = -\frac{\partial H}{\partial I} = \lambda_3 b(t)(\mu_h + \gamma_h + \alpha_h) - \lambda_4\gamma_\upsilon(1 - I_\upsilon - E_\upsilon)b(t) - 1,$$

$$\lambda_4' = -\frac{\partial H}{\partial E_\upsilon} = \lambda_4(b(t)\gamma_\upsilon I + \mu_\upsilon + \delta_\upsilon) - \lambda_5\delta_\upsilon$$

$$\lambda_5' = -\frac{\partial H}{\partial I_\upsilon} = (\lambda_1 - \lambda_2)\alpha_h(1-a(t))S + \lambda_4\gamma_\upsilon I b(t) + \lambda_5\mu_\upsilon$$

With the transversality conditions $\lambda_1(t_f) = \lambda_2(t_f) = \lambda_3(t_f) = \lambda_4(t_f) = \lambda_5(t_f) = 0$, and the optimize control (a^*, b^*) is given by

$$a^* = \min\left\{1, \max\left\{0, \frac{((\lambda_2 - \lambda_1)(\alpha_h I_\upsilon)S^*}{B_1}\right\}\right\}$$

$$b^* = \min\left\{1, \max\left\{0, \frac{(\lambda_3(\mu_h + \gamma_h + \alpha_h) - \lambda_4\gamma_\upsilon(1 - I^*_\upsilon - E^*_\upsilon))I^*}{B_2}\right\}\right\}.$$

Proof.

The existence of optimal control can be proved by using the results from [9]. The adjoint equations and transversality conditions can be obtained by using Pontryagin's Maximum Principle such that

$$\lambda_1' = -\frac{\partial H}{\partial S} = \lambda_1(\mu_h + \rho + \alpha_h(1-a(t))I_\upsilon) - \lambda_2((\alpha_h I_\upsilon)(1-a(t)) + \rho) \ ,$$

$$\lambda_2' = -\frac{\partial H}{\partial E} = \lambda_2(\mu_h + \varphi_h) - \lambda_3\varphi_h - 1 \ ,$$

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$$\lambda_3' = -\frac{\partial H}{\partial I} = \lambda_3 b(t)\left(\mu_h + \gamma_h + \alpha_h\right) - \lambda_4 \gamma_\upsilon \left(1 - I_\upsilon - E_\upsilon\right) b(t) - 1,$$
$$\lambda_4' = -\frac{\partial H}{\partial E_\upsilon} = \lambda_4 (b(t)\gamma_\upsilon I + \mu_\upsilon + \delta_\upsilon) - \lambda_5 \delta_\upsilon$$
$$\lambda_5' = -\frac{\partial H}{\partial I_\upsilon} = (\lambda_1 - \lambda_2)\alpha_h (1 - a(t))S + \lambda_4 \gamma_\upsilon I b(t) + \lambda_5 \mu_\upsilon,$$

the optimal control pair $\left(a^*, b^*\right)$ are obtained by finding the derivative of the Hamiltonian equation with respect to the control variables, equating to zero, and solving equation. Then we get;

$$\frac{\partial H}{\partial a} = B_1 a(t) + \lambda_1 \alpha_h S I_\upsilon - \lambda_2 (\alpha_h I_\upsilon) S .$$

Let $B_1 a(t) + \lambda_1 \alpha_h S I_\upsilon - \lambda_2 (\alpha_h I_\upsilon) S = 0$

Then the optimal value for a is;

$$a^* = \frac{(\lambda_2 - \lambda_1)(\alpha_h I_\upsilon) S^*}{B_1}$$

And $\dfrac{\partial H}{\partial b} = \dfrac{B_2 b(t) - \lambda_3 (\mu_h + \gamma_h + \alpha_h) I + \lambda_4 \gamma_\upsilon (1 - I_\upsilon - E_\upsilon) I}{B_2}$,

Let $\dfrac{B_2 b(t) - \lambda_3 (\mu_h + \gamma_h + \alpha_h) I + \lambda_4 \gamma_\upsilon (1 - I_\upsilon - E_\upsilon) I}{B_2} = 0$

Hence the optimal value for b is;

$$b^* = \frac{(\lambda_3 (\mu_h + \gamma_h + \alpha_h) - \lambda_4 \gamma_\upsilon (1 - I_\upsilon - E_\upsilon)) I^*}{B_2}$$

By the bounds in U of the control, the optimal control pair $\left(a^*, b^*\right)$ is given by

$$a^* = \min\left\{1, \max\left\{0, \frac{(\lambda_2 - \lambda_1)(\alpha_h I_\upsilon) S^*}{B_1}\right\}\right\}$$
$$b^* = \min\left\{1, \max\left\{0, \frac{(\lambda_3 (\mu_h + \gamma_h + \alpha_h) - \lambda_4 \gamma_\upsilon (1 - I^*_\upsilon - E^*_\upsilon)) I^*}{B_2}\right\}\right\}. \qquad (24)$$

For supporting analytic results we need to resolve the optimal control model numerically.

NUMERICAL SIMULATIONS

In this section we present the results obtained by solving numerically from the following optimality system;

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$$\frac{dS}{dt} = \mu_h(1-S) - \rho S - \alpha_h(1-\min\left\{1, \max\left\{0, \frac{(\lambda_2-\lambda_1)(\alpha_h I_\upsilon)S}{B_1}\right\}\right\})SI_\upsilon$$

$$\frac{dE}{dt} = (\alpha I_\upsilon)(1-\min\left\{1, \max\left\{0, \frac{(\lambda_2-\lambda_1)(\alpha_h I_\upsilon)S}{B_1}\right\}\right\})S + \rho S - (\mu_h + \varphi_h)E$$

$$\frac{dI}{dt} = \varphi_h E - (\mu_h + \gamma_h + \alpha_h)(\min\left\{1, \max\left\{0, \frac{(\lambda_3(\mu_h+\gamma_h+\alpha_h) - \lambda_4\gamma_\upsilon(1-I_\upsilon-E_\upsilon))I}{B_2}\right\}\right\})I$$

$$\frac{dE_\upsilon}{dt} = \gamma_\upsilon(1-I_\upsilon-E_\upsilon)I(\min\left\{1, \max\left\{0, \frac{(\lambda_3(\mu_h+\gamma_h+\alpha_h) - \lambda_4\gamma_\upsilon(1-I_\upsilon-E_\upsilon))I}{B_2}\right\}\right\}) - (\mu_\upsilon + \delta_\upsilon)E_\upsilon$$

$$\frac{dI_\upsilon}{dt} = \delta_\upsilon E_\upsilon - \mu_\upsilon I_\upsilon$$

$$\lambda_1' = -\frac{\partial H}{\partial S} = \lambda_1(\mu_h + \rho + \alpha_h(1-\min\left\{1, \max\left\{0, \frac{(\lambda_2-\lambda_1)(\alpha_h I_\upsilon)S}{B_1}\right\}\right\})I_\upsilon)$$

$$-\lambda_2\left(\alpha_h I_\upsilon\right)(1-\min\left\{1, \max\left\{0, \frac{(\lambda_2-\lambda_1)(\alpha_h I_\upsilon)S}{B_1}\right\}\right\} + \rho)$$

$$\lambda_2' = -\frac{\partial H}{\partial E} = \lambda_2\left(\mu_h + \varphi_h\right) - \lambda_3\varphi_h - 1$$

$$\lambda_3' = -\frac{\partial H}{\partial I} = \lambda_3(\min\left\{1, \max\left\{0, \frac{(\lambda_3(\mu_h+\gamma_h+\alpha_h) - \lambda_4\gamma_\upsilon(1-I_\upsilon-E_\upsilon))I}{B_2}\right\}\right\})\left(\mu_h + \gamma_h + \alpha_h\right)$$

$$-\lambda_4\gamma_\upsilon\left(1-I_\upsilon-E_\upsilon\right)(\min\left\{1, \max\left\{0, \frac{(\lambda_3(\mu_h+\gamma_h+\alpha_h) - \lambda_4\gamma_\upsilon(1-I_\upsilon-E_\upsilon))I}{B_2}\right\}\right\}) - 1$$

$$\lambda_4' = -\frac{\partial H}{\partial E_\upsilon} = \lambda_4((\min\left\{1, \max\left\{0, \frac{(\lambda_3(\mu_h+\gamma_h+\alpha_h) - \lambda_4\gamma_\upsilon(1-I_\upsilon-E_\upsilon))I}{B_2}\right\}\right\})\gamma_\upsilon I + \mu_\upsilon + \delta_\upsilon) - \lambda_5\delta_\upsilon$$

$$\lambda_5' = -\frac{\partial H}{\partial I_\upsilon} = (\lambda_1 - \lambda_2)\alpha_h(1-\min\left\{1, \max\left\{0, \frac{(\lambda_2-\lambda_1)(\alpha_h I_\upsilon)S}{B_1}\right\}\right\})S$$

$$+\lambda_4\gamma_\upsilon I(\min\left\{1, \max\left\{0, \frac{(\lambda_3(\mu_h+\gamma_h+\alpha_h) - \lambda_4\gamma_\upsilon(1-I_\upsilon-E_\upsilon))I}{B_2}\right\}\right\}) + \lambda_5\mu_\upsilon$$

With $S(0)=S_0, E(0)=E_0, I(0)=I_0, E_\upsilon(0)=E_{\upsilon_0} and I_\upsilon(0)=I_{\upsilon_0}$ and $\lambda_i\left(t_f\right)=0, \left(i=1,2,3,4,5\right)$

Since ,there were initial condition for the state variables and terminal conditions for the adjoints and the optimality system is two-point boundary value problem, with separated boundary conditions at $t=0$ and t_f. Then we use the semi-implicit finite difference method to solve the optimality system (20).

We partition the interval $\left[t_0, t_f\right]$ at the point $t_i = t_0 + ih\left(i=0,1,2,...,n\right)$, where h is the time step such that $t_n = t_f$. And we define the state and adjoint variable $S\left(t\right), E(t), I(t), E_\upsilon(t), I_\upsilon(t), \lambda_1, \lambda_2, \lambda_3, \lambda_4, \lambda_5$ and the control a and b in terms of nodal points $S_i, E_i, I_i, E_{\upsilon_i}, I_{\upsilon_i}, \lambda_1^i, \lambda_2^i, \lambda_3^i, \lambda_4^i, \lambda_5^i, a^i$ and b^i. Then we use a combination of forward and backward difference approximation as follows:

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$$\frac{S_{i+1}-S_i}{h}=\mu_h\left(1-S_{i+1}\right)-\rho S_{i+1}-\alpha_h S_{i+1}(1-a^i)I_{\upsilon_i}$$
$$\frac{E_{i+1}-E_i}{h}=\left(\alpha_h I_{\upsilon_i}\right)(1-a^i)S_{i+1}+\rho S_{i+1}-\left(\mu_h+\varphi_h\right)E_{i+1}$$
$$\frac{I_{i+1}-I_i}{h}=\varphi_h E_{i+1}-\left(\mu_h+\gamma_h+\alpha_h\right)b^i I_{i+1}$$
$$\frac{E_{\upsilon_{i+1}}-E_{\upsilon_i}}{h}=\gamma_\upsilon(1-I_{\upsilon_i}-E_{\upsilon_{i+1}})I_{i+1}b^i-(\mu_\upsilon+\delta_\upsilon)E_{\upsilon_{i+1}}$$
$$\frac{I_{\upsilon_{i+1}}-I_{\upsilon_i}}{h}=\delta_\upsilon E_{\upsilon_{i+1}}-\mu_\upsilon I_{\upsilon_{i+1}}$$
By using above technique, we approximate the time derivative of the adjoint variables by their first-order backward-difference as the following;
$$\frac{\lambda_1^{n-i}-\lambda_1^{n-i-1}}{h}=\lambda_1^{n-i-1}(\mu_h+\rho+\alpha_h(1-a^i)I_{\upsilon_{i+1}})-\lambda_2^{n-i}((\alpha_h I_{\upsilon_{i+1}})(1-a^i)+\rho),$$
$$\frac{\lambda_2^{n-i}-\lambda_2^{n-i-1}}{h}=\lambda_2^{n-i-1}\left(\mu_h+\varphi_h\right)-\lambda_3^{n-i}\varphi_h-1$$
$$\frac{\lambda_3^{n-i}-\lambda_3^{n-i-1}}{h}=\lambda_3^{n-i-1}b^i\left(\mu_h+\gamma_h+\alpha_h\right)-\lambda_4^{n-i}\gamma_\upsilon\left(1-I_{\upsilon_{i+1}}-E_{\upsilon_{i+1}}\right)b^i-1$$
$$\frac{\lambda_4^{n-i}-\lambda_4^{n-i-1}}{h}=\lambda_4^{n-i-1}(b^i\gamma_\upsilon I_{i+1}+\mu_\upsilon+\delta_\upsilon)-\lambda_5^{n-i}\delta_\upsilon$$
$$\frac{\lambda_5^{n-i}-\lambda_5^{n-i-1}}{h}=(\lambda_1^{n-i}-\lambda_2^{n-i})\alpha_h(1-a^i)S_{i+1}+\lambda_4^{n-i}\gamma_\upsilon I_{i+1}b^i+\lambda_5^{n-i-1}\mu_\upsilon$$
The algorithm for the approximation method to obtain the optimal control as follows;

Algorithm

Step 1: $S\left(0\right)=S_0, E\left(0\right)=E_0, I\left(0\right)=I_0, E_\upsilon(0)=E_{\upsilon_0}, I_\upsilon(0)=I_{\upsilon_0}, \lambda_i\left(t_f\right)=0\left(i=1,2,3,4,5\right)$ and $a(0)=b(0)=0$

Step 2: For $i=0,\ldots,\mathrm{n}-1$, do
$$S_{i+1}=\frac{S_i+h\mu_h}{1+h(\mu_h+\rho+\alpha_h I_{\upsilon_i}(1-a^i))}$$
$$E_{i+1}=\frac{h(\alpha_h I_{\upsilon_i}(1-a^i)+\rho)S_{i+1}+E_i}{1+h(\mu_h+\varphi_h)}$$
$$I_{i+1}=\frac{h\varphi_h E_{i+1}+I_i}{1+h(\mu_h+\gamma_h+\alpha_h)b^i}$$
$$E_{\upsilon_{i+1}}=\frac{h\gamma_\upsilon(1-I_{\upsilon_{i+1}})I_{i+1}b^i+E_{\upsilon_i}}{h(\gamma_\upsilon I_{i+1}b^i+\mu_\upsilon+\delta_\upsilon)+1}$$
$$I_{\upsilon_{i+1}}=\frac{h\delta_\upsilon E_{\upsilon_{i+1}}+I_{\upsilon_i}}{1+h\mu_\upsilon}$$

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$$\lambda_1^{n-i-1} = \frac{\lambda_1^{n-i} + h((\alpha_h I_{\upsilon_{i+1}})(1-a^i) + \rho)\lambda_2^{n-i}}{h[\mu_h + \rho + \alpha_h(1-a^i)I_{\upsilon_{i+1}}] + 1}$$

$$\lambda_2^{n-i-1} = \frac{\lambda_2^{n-i} + h\left(1 + \varphi_h \lambda_3^{n-i}\right)}{1 + h\left(\mu_h + \varphi_h\right)}$$

$$\lambda_3^{n-i-1} = \frac{\lambda_3^{n-i} + h(1 + \lambda_4^{n-i}\gamma_\upsilon b^i(1 - I_{\upsilon_{i+1}} - E_{\upsilon_{i+1}}))}{1 + hb^i(\mu_h + \gamma_h + \alpha_h)}$$

$$\lambda_4^{n-i-1} = \frac{\lambda_4^{n-i} + h\lambda_5^{n-i}\delta_\upsilon}{1 + h(b^i\gamma_\upsilon I_{i+1} + \mu_\upsilon + \delta_\upsilon)}$$

$$\lambda_5^{n-i-1} = \frac{\lambda_5^{n-i} + h((\lambda_2^{n-i} - \lambda_1^{n-i})(1-a^i)\alpha_h S_{i+1} - \lambda_4^{n-i}\gamma_\upsilon I_{i+1} b^i)}{1 + h\mu_\upsilon}$$

$$M^{i+1} = \frac{(\lambda_2^{n-i-1} - \lambda_1^{n-i-1})(\alpha_h I_{\upsilon+1})S_{i+1}}{B_1}$$

$$T^{i+1} = \frac{(\lambda_3^{n-i-1}(\mu_h + \gamma_h + \alpha_h) - \lambda_4^{n-i-1}\gamma_\upsilon(1 - I_{\upsilon_{i+1}} - E_{\upsilon_{i+1}}))I_{i+1}}{B_2}$$

$$a^{i+1} = \min\left\{1, \max\{0, M^{i+1}\}\right\}$$

$$b^{i+1} = \min\left\{1, \max\{0, \mathrm{T}^{i+1}\}\right\}$$

End for

Step 3:

$S^*(t_i) = S_i, E^*(t_i) = E_i, I^*(t_i) = I_i, E_\upsilon^*(t_i) = E_{\upsilon_i}, I_\upsilon^*(t_i) = I_{\upsilon_i} a^*(t_i) = a^i$ and $b^*(t_i) = b^i$

End for

The simulations at endemic state were carried out using the following values taken from table1, with initial condition; $S(0) = \frac{99900}{100000}, E(0) = \frac{10500}{100000}, I(0) = \frac{24.13}{100000}, E_\upsilon(0) = 0.01 and I_\upsilon(0) = 0.1$ and results show below;

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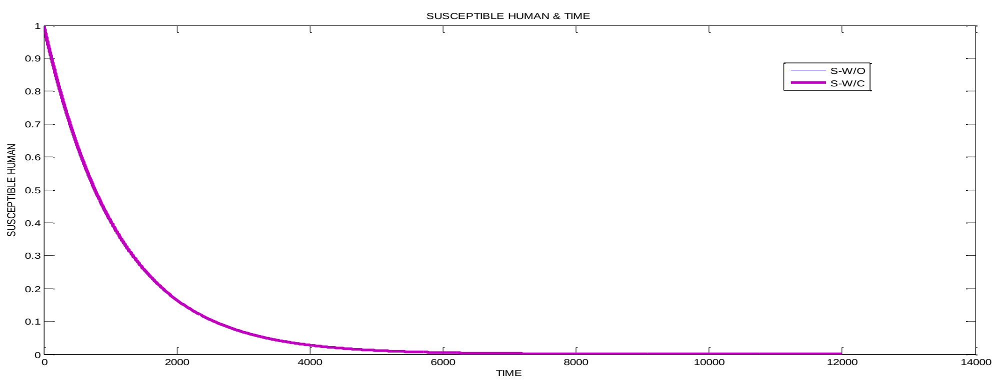
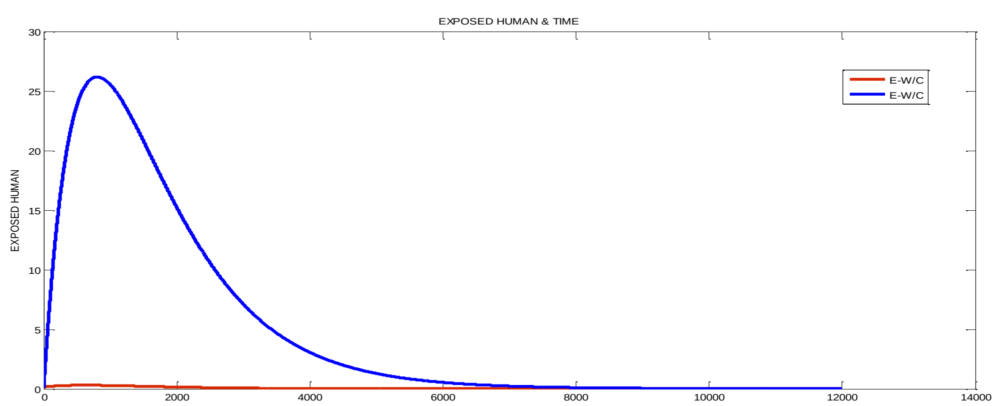
Table1 Parameters values used in numerical simulation at endemic state.

Parameters	Description	Value
μ_h	Death rate of human populations	0.0000391 day^{-1}
μ_υ	Death rate of vector populations	0.071 day^{-1}
ρ	Percentage of infected mosquitoes	0.09
δ_υ	Proportional rates of mosquitoes exposed to the virus infection	0.1428
γ_h	the duration of infection in the body	0.329
$\beta_h c$	the sufficient rate of correlation from human to vector	0.75
$\beta_\upsilon c = \gamma_\upsilon$	the sufficient rate of correlation from vector to human	0.375
N_h	Number of human populations	100000
α_h	Death rate caused by denque itself	0.000003
φ_h	Proportional rates for people exposed to the denque virus infection	0.167

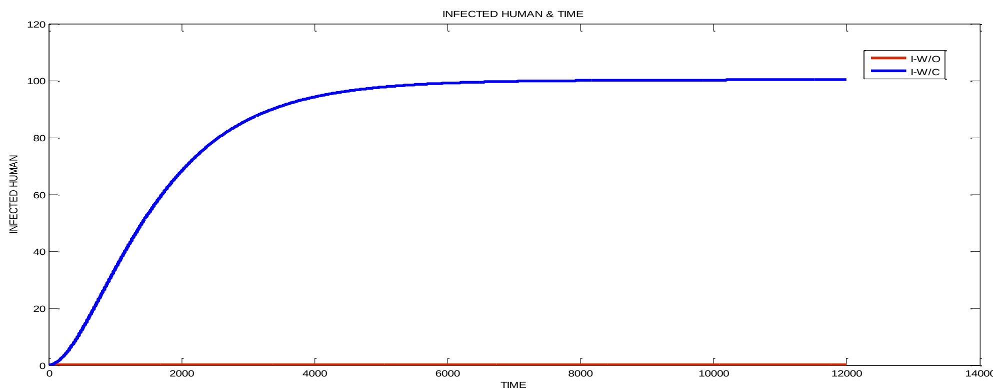


Figures 1 Represent time series of susceptible individuals (S) with and without controls. It's show the different between the number of susceptible individuals (S) with and without controls are small positive values, it means that the number of susceptible should be decrease after control.



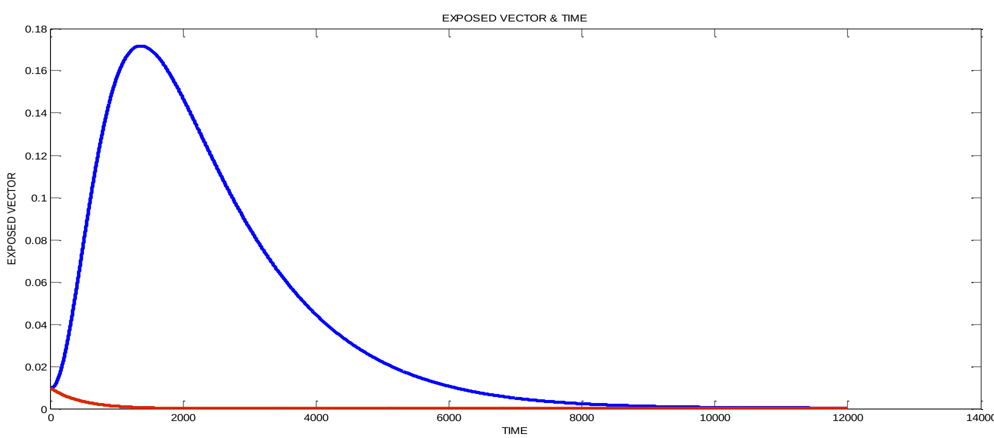
Figures 2 Represent time series of exposed individuals (E) with and without controls.

It's show the number of exposed individuals (S) with controls are increased rapidly.



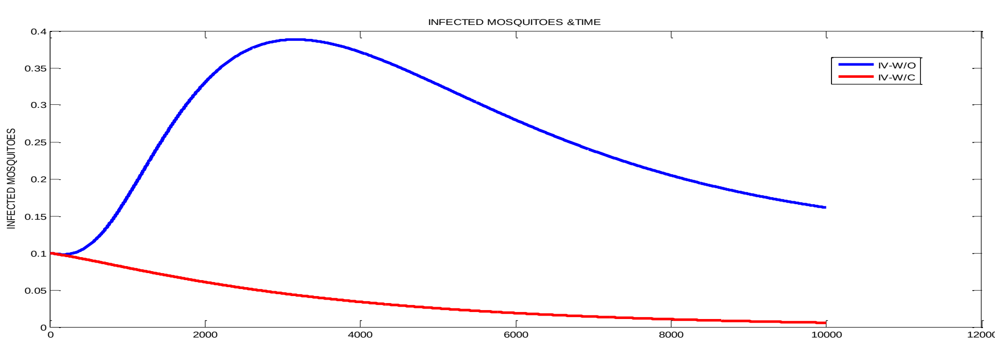
Figures 3 Represent time series of infectious individuals (I) with and without controls.

It's show that the number of infected individuals (S) with controls are decreased rapidly.



Figures 4 Represent time series of exposed vector (E_υ) with and without controls.

It's show the number of exposed vector (E_υ) with controls are increased rapidly.



Figures 5 Represent time series of infected vector (I_υ) with and without controls.

It's show the number of infected vector (I_υ) with controls are decreased rapidly.

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CONCLUSION

In this paper, an SEIR model for transmission of denque fever with the were proposed and analyzed. To reduce the contract between the susceptible human and the infected vector and the other, minimizing the population of the infected human. The optimal control theory has been applied. By using the Pontryagin's maximum principle, the explicit expression of the optimal controls was obtained. Simulation results indicate that the difference between the numbers of susceptible individuals (S), exposed individuals (E) and infectious individuals (I),exposed vector (E_υ) and infectious vector (I_υ) compare with and without control. After control are decreased rapidly.

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